ME 423: FLUIDS ENGINEERING

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Hydraulics of Pipeline Systems

(Transient System / Unsteady flow)

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Transient System / Unsteady flow in Pipelines

Unsteady, or transient flows in pipelines traditionally have been associated with **hydropower piping and with long water and oil pipeline delivery systems**. However, the application has broadened in recent years to include **hydraulic control system operation, events that take place in power plant piping networks, fluid–structure interaction in liquid-filled piping, and pulsatile blood flow.**

There are two general categories of unsteady flow. The **first type is referred to as quasi-steady flow, is characterized by the absence of elastic effects on the flow behavior**. In such a flow the variation of discharges and pressures with time is gradual, and over short time intervals the flow appears to be steady.

For the unsteadiness to occur, some type of excitation to the system is necessary.
Representative excitations are **valve opening or closure, pump or turbine operations, pipe rupture or break, and cavitation events**.

Transient System / Unsteady flow in Pipelines

If inertial **effects are significant but pipe and fluid compressibility effects are relatively minor or negligible**, then we have a transient flow which we will refer to as a **rigid-column flow**. Unsteady incompressible flow in an inelastic pipe results in the phenomenon called **surging**.

Typical examples are the drawdown of a reservoir, the draining of a large tank, or the variation in demand in a water distribution system over a 24-hour period.

If in addition (to inertia) if we need to consider the **elastic effects of the fluid (compressibility) and pipe (elastic pipe)** in order to obtain an accurate characterization of the transient, we will call this a **water hammer** condition.

Incompressible Flow in an Inelastic Pipe (Surging)

Consider a single horizontal pipe of length L and diameter D (Fig. 11.8). The upstream end of the pipe is connected to a reservoir and a valve is situated at the downstream end. The upstream piezometric head (HGL) is H_1, and downstream of the valve the piezometric head is H_3. Note that both H_1 and H_3 are constant time-independent reservoir elevations.

The friction factor f is assumed constant, and the loss coefficient for the valve is K. There are a number of possible excitations that could be considered, but we will only look at the situation where initially there is a steady velocity V_0, the valve is then instantaneously opened to a new position, and subsequently the flow accelerates, increasing to a new steady-state velocity V_{ss}.

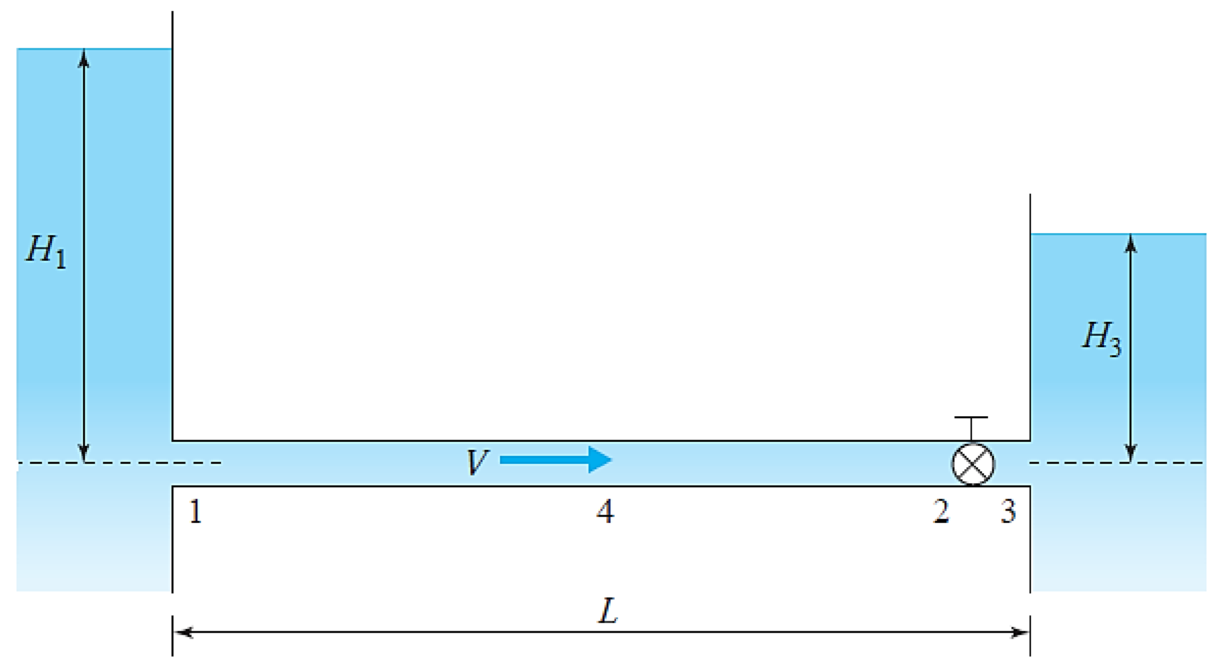


Fig. 11.8 Horizontal pipe with a valve at the downstream end.

Incompressible Flow in an Inelastic Pipe

In Fig. 11.8, define a control volume for the liquid in the pipe between locations 1 and 2, whose mass is ρAL. Note that location 2 is upstream of the valve. At the two ends of the control volume the pressures are p_1 and p_2, respectively, and on the surface the wall shear stress is τ_0. The conservation of momentum for that liquid volume is given by

$$A(p_1 - p_2) - \tau_0 \pi D L = \rho A L \frac{dV}{dt} \qquad (11.5.1)$$

Without serious consequences we can assume steady-flow conditions across the valve from location 2 to 3, and utilize the energy equation to provide

$$p_2 = p_3 + K \frac{\rho V^2}{2} \qquad (11.5.2)$$

It is reasonable to assume that the Darcy–Weisbach friction factor based on steady-state flow can be employed without undue error.

$$\tau_0 = \frac{\rho f V^2}{8} \qquad (11.5.3)$$

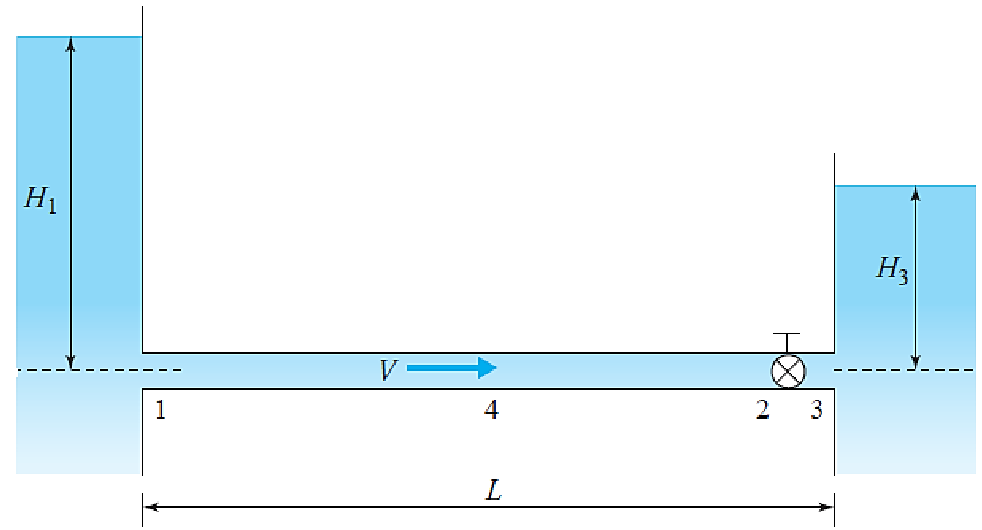


Fig. 11.8 Horizontal pipe with a valve at the downstream end.

Incompressible Flow in an Inelastic Pipe

Substituting Eqs. 11.5.2 and 11.5.3 into Eq. 11.5.1, dividing by the mass of the liquid column, and recognizing that $p_1 - p_3 = \rho g(H_1 - H_3)$, since the velocity heads are assumed to be negligible, there results

$$\frac{dV}{dt} + \left(\frac{f}{D} + \frac{K}{L}\right)\frac{V^2}{2} - g\,\frac{H_1 - H_3}{L} = 0 \qquad (11.5.4)$$

Equation 11.5.4 is the relation that represents incompressible unsteady flow in the pipe. The initial condition at t = 0 is a given velocity $V = V_0$.

When the **final steady-state condition** is attained, $dV/dt = 0$, and that steady-state velocity, designated as V_{ss}, can be obtained by setting the derivative to zero in Eq. 11.5.4:

$$V_{ss} = \sqrt{\frac{2g(H_1 - H_3)}{fL/D + K}} \qquad (11.5.5)$$

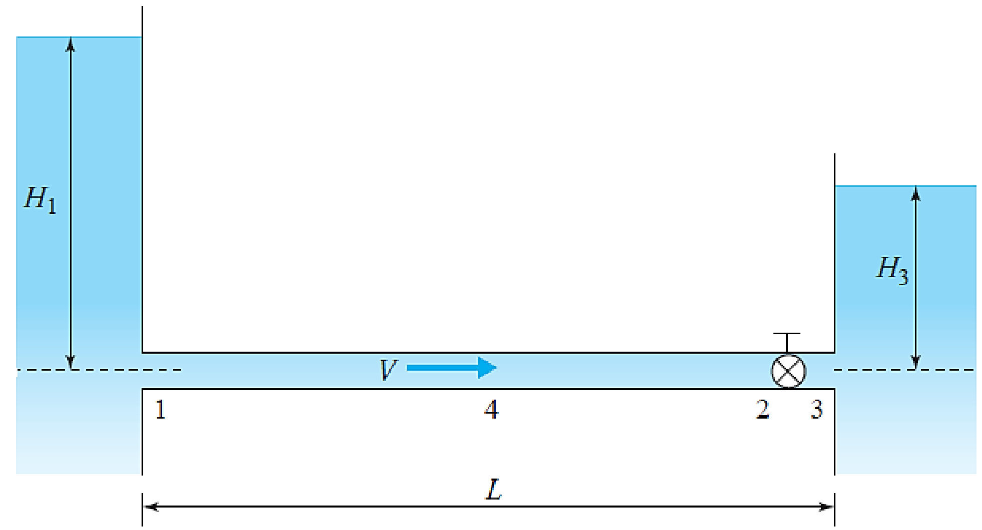


Fig. 11.8 Horizontal pipe with a valve at the downstream end.

Incompressible Flow in an Inelastic Pipe

Substituting Eq. 11.5.5 into Eq. 11.5.4, separating variables, and expressing the result in integral form, one has

$$\int_0^t dt = \frac{V_{ss}^2 L}{g(H_1 - H_3)} \int_{V_0}^{V} \frac{dV}{V_{ss}^2 - V^2} \qquad (11.5.6)$$

Upon integration, the resulting relation defines the velocity V relative to time t after the valve excitation:

$$t = \frac{V_{ss} L}{2g(H_1 - H_3)} \ln \frac{(V_{ss} + V)(V_{ss} - V_0)}{(V_{ss} - V)(V_{ss} + V_0)} \qquad (11.5.7)$$

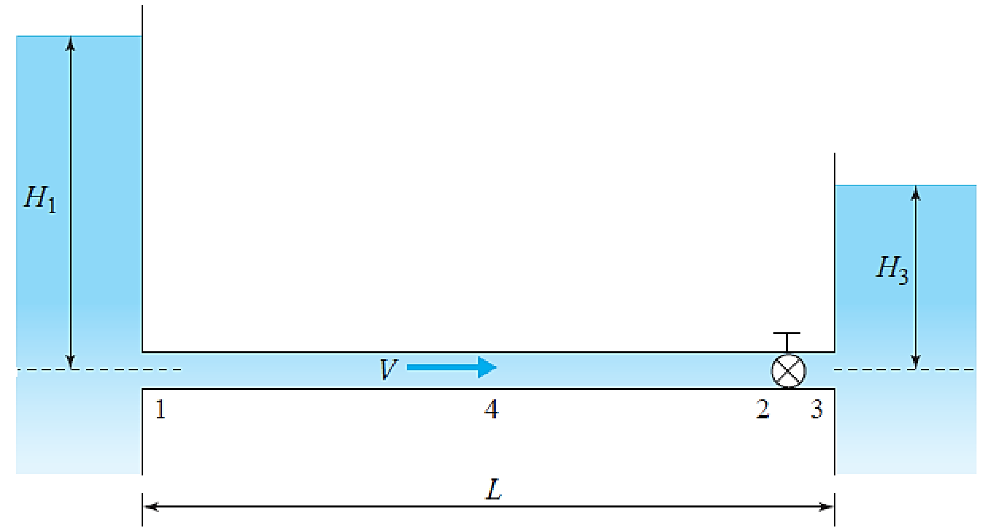


Fig. 11.8 Horizontal pipe with a valve at the downstream end.

There are some significant features to the solution. First, by studying Eq. 11.5.7 one concludes that infinite time is required to reach the steady-state velocity V_{ss}.

In reality, V_{ss} will be reached some time sooner, because the losses have not been completely accounted for. However, it is possible to determine the time when a percentage, for example 99%, of V_{ss} has been reached that would be adequate for engineering purposes.

In addition, it is possible for the fluid to be initially at rest, that is, V_0 = 0. This solution is based on the assumptions that both liquid compressibility and pipe elasticity are ignored.

Example 11.10

A horizontal pipe 1000 m in length, with a diameter of 500 mm, and a steady velocity of 0.5 m/s, is suddenly subjected to a new piezometric head differential of 20 m when the downstream valve suddenly opens and its coefficient changes to K = 0.2. Assuming a friction factor of f = 0.02, determine the final steady-state velocity, and the time when the actual velocity is 75% of the final value.

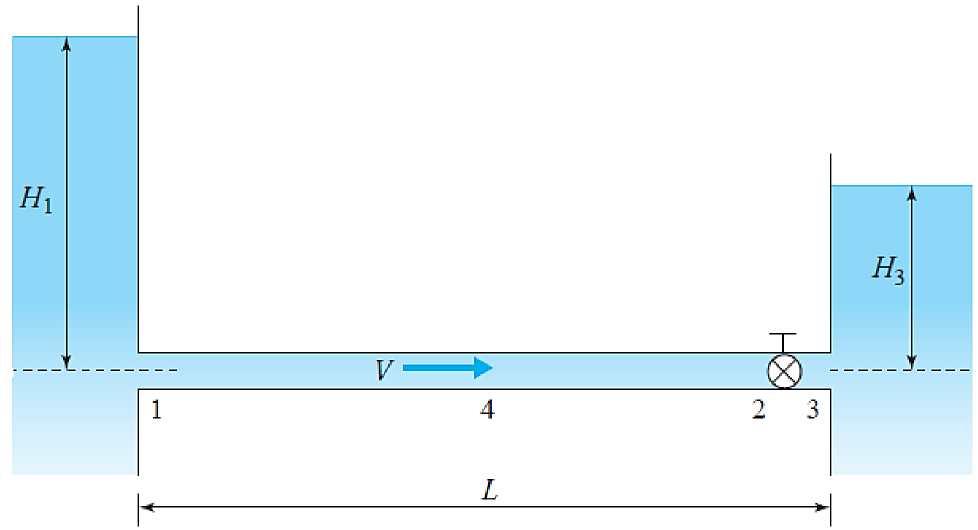


Fig. 11.8 Horizontal pipe with a valve at the downstream end.

Solution

Given are $L = 1000$ m, $D = 0.5$ m, $V_0 = 0.5$ m/s, $f = 0.02$, and $K = 0.2$. Setting $H_1 - H_3 = 20$ m, the final steady-state velocity V_{ss} is found by direct substitution into Eq. 11.5.5:

$$V_{ss} = \sqrt{\frac{2 \times 9.81 \times 20}{0.02 \times 1000/0.5 + 0.2}} = 3.12 \text{ m/s}$$

The velocity that is 75% of V_{ss} is $V = 0.75 \times 3.12 = 2.34$ m/s. Then, using Eq. 11.5.7, the time corresponding to that velocity is

$$t = \frac{3.12 \times 1000}{2 \times 9.81 \times 20} \ln \frac{(3.12 + 2.34) \times (3.12 - 0.5)}{(3.12 - 2.34) \times (3.12 + 0.5)} = 12.9 \text{ s}$$

Hence the final steady-state velocity is 3.12 m/s, and the time when 75% of that velocity is attained is approximately 13 s.

(continuation)

Also compute the time required to reach the velocity to 25%, 50%, 90%, 95% and 99% of final steady state velocity.

Plot the velocity in the pipe as function of time.

Incompressible Flow in an Inelastic Pipe

Problem 11.46

A Gasoline is supplied by gravity without pumping from a storage tank through a 800-m-long 50-mm diameter nearly horizontal pipe into a tanker truck. There is a quick-acting valve at the end of the pipe. The difference in elevations of gasoline between the reservoir and the truck tank is 8 m. Initially, the valve is partially closed so that K = 275. Then the operator decides to increase the discharge by opening the valve quickly to the position where K = 5.

Assuming an incompressible fluid and an inelastic piping system, determine the new steady-state discharge and the time it takes to reach 95% of that value. Assume that f = 0.015.

Solution:

First compute the initial velocity V_0, with H_1-H_3 = 8 m, and K_0 = 275:

$$V_0 = \sqrt{\frac{2\times 9.81\times 8}{\frac{0.015\times 800}{0.05}+275}} = 0.552 \text{ m/s}$$

The final steady-state velocity is, with K_{ss} = 5:

$$V_{SS} = \sqrt{\frac{2\times 9.81\times 8}{\frac{0.015\times 800}{0.05}+5}} = 0.800 \text{ m/s}$$

Incompressible Flow in an Inelastic Pipe

The steady-state discharge is Q = (π/4)×0.052 ×0.800 = 0.00157 m^3 /s, and the time to reach 95% of that value is

$$V = 0.95V_{ss} = 0.95\times 0.800 = 0.760 \text{ m/s}$$

$$\therefore t = \frac{0.800\times 800}{2\times 9.81\times 8}\ln\left[\frac{(0.800+0.760)(0.800-0.552)}{(0.800-0.760)(0.800+0.552)}\right] = \underline{8.03 \text{ s}}$$

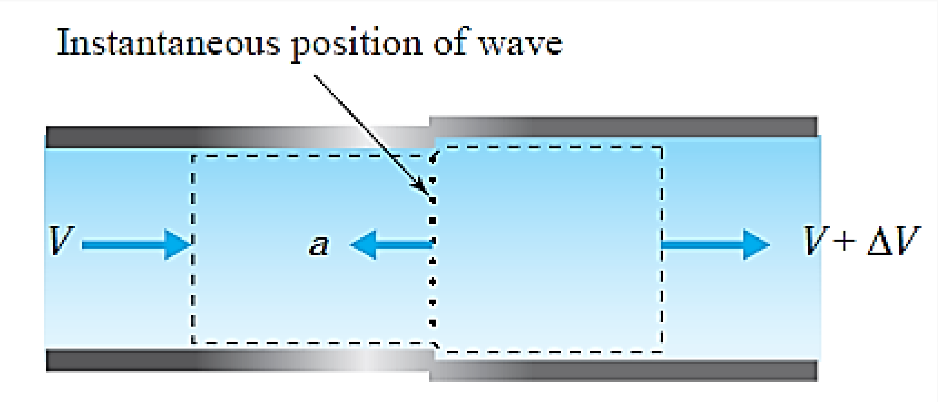
Compressible Flow in an Elastic Pipe (water hammer)

There are situations where the liquid is not incompressible and the piping is not rigid.

The interaction between changes in momentum and applied forces causes the liquid to slightly compress and the pipe material to experience very small deformations. When this occurs, significant pressure changes can take place, and the phenomenon is termed **water hammer**.

Water hammer is accompanied by **pressure and velocity perturbations traveling at very high velocities**, close to the speed of sound in the liquid. The resulting wave action occurs at relatively high frequencies.

Consider a horizontal pipe where the valve is closed so rapidly that elastic effects caused water hammer to occur. The movement of the valve will cause an acoustic, or pressure, wave with speed *a* to propagate upstream. A control volume of an incremental section of liquid contained in the pipe is shown below, where the pressure wave at a given instant is located.



Compressible Flow in an Elastic Pipe

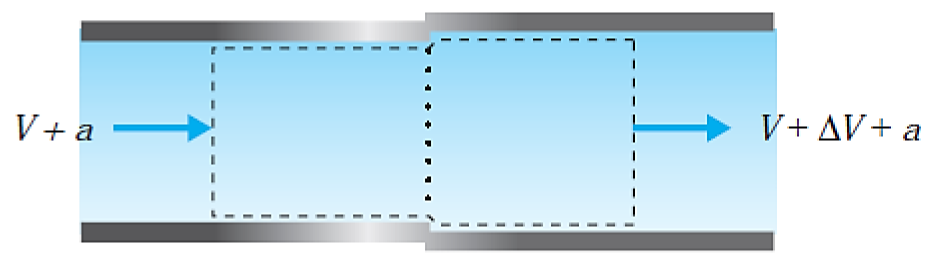
The presence of the wave implies that an unsteady flow is taking place within the control volume; at the entrance the velocity is V, and at the exit it is V + ΔV. Steady-state conservation laws can be applied if the wave front is made to appear stationary by an observer moving with the wave speed. The entrance velocity is now V + a, and at the exit it is V + ΔV + a.

The pressure, pipe area, and density at the entrance are p, A, and ρ, respectively. Due to the passage of the pressure wave, at the exit the pressure, pipe area, and liquid density are altered to p + Δp, A + ΔA, and ρ + Δρ, where p, A, and ρ are the respective changes in pressure, area, and density.

Applying the conservation of mass across the control volume,

we find that

$$0 = (\rho + \Delta\rho)(V + \Delta V + a)(A + \Delta A) - \rho(V + a)A \qquad (11.5.8)$$



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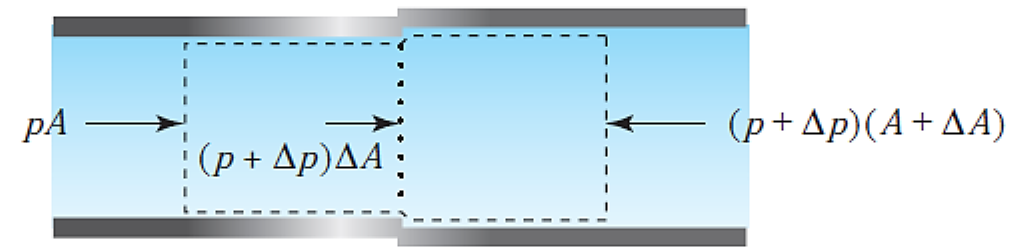


(c)

Compressible Flow in an Elastic Pipe

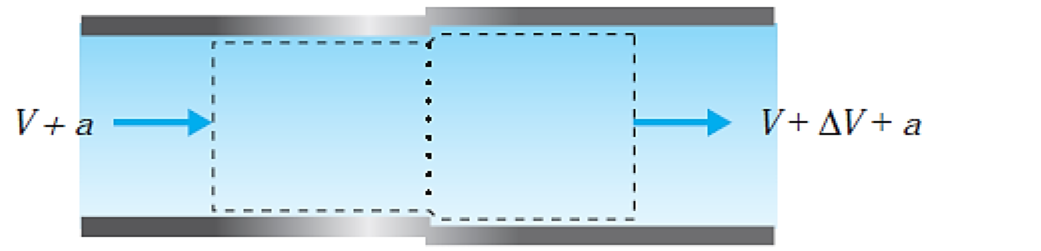
Neglecting frictional and gravitational forces, only pressure forces act on the control volume in the direction of flow, as shown in the Fig. The conservation of momentum across the control volume is

$$pA + (p + \Delta p)\,\Delta A - (p + \Delta p)(A + \Delta A) = \rho A(V + a)[V + \Delta V + a - (V + a)] \qquad (11.5.9)$$

Equations 11.5.8 and 11.5.9 are expanded, and the terms containing factors of Δ^2 and Δ^3 are dropped, since they are much smaller in magnitude than the remaining ones. Then Eqs. 11.5.8 and 11.5.9 become

$$\rho A \Delta V + (V + a)(A\Delta\rho + \rho\Delta A) = 0 \qquad (11.5.10)$$

$$-A\Delta p = \rho A(V + a)\,\Delta V \qquad (11.5.11)$$



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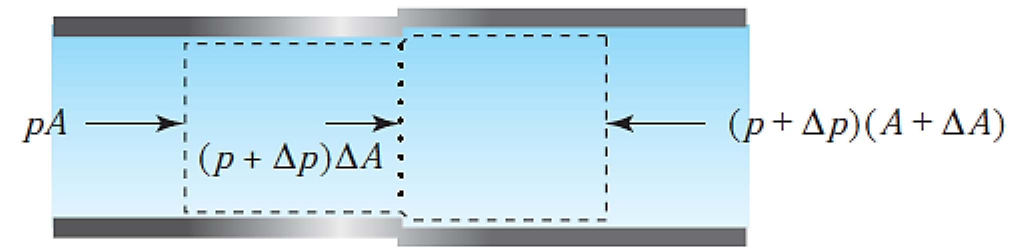


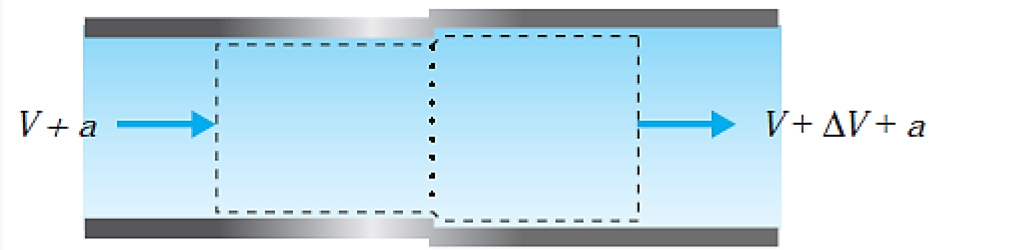
(c)

Compressible Flow in an Elastic Pipe

In nearly all situations V << a, so that Eq. 11.5.11 becomes

$$\Delta p = -\rho a \Delta V \qquad (11.5.12)$$

Equation 11.5.12, called the **Joukowsky equation**, relates the pressure change to the pressure wave speed and the change in velocity. Note that a velocity reduction (a negative ΔV) produces a pressure rise (a positive Δp), and a positive ΔV yields a negative Δp.



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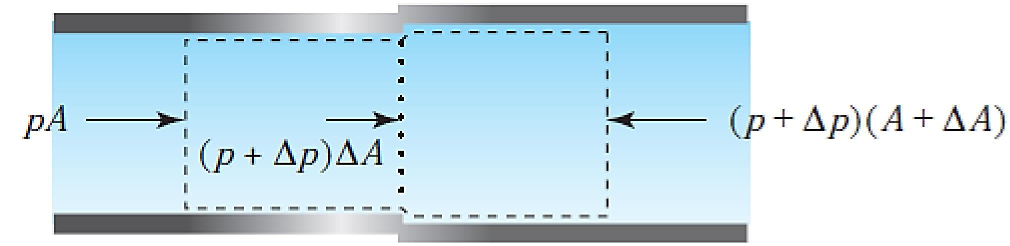


(c)

Compressible Flow in an Elastic Pipe

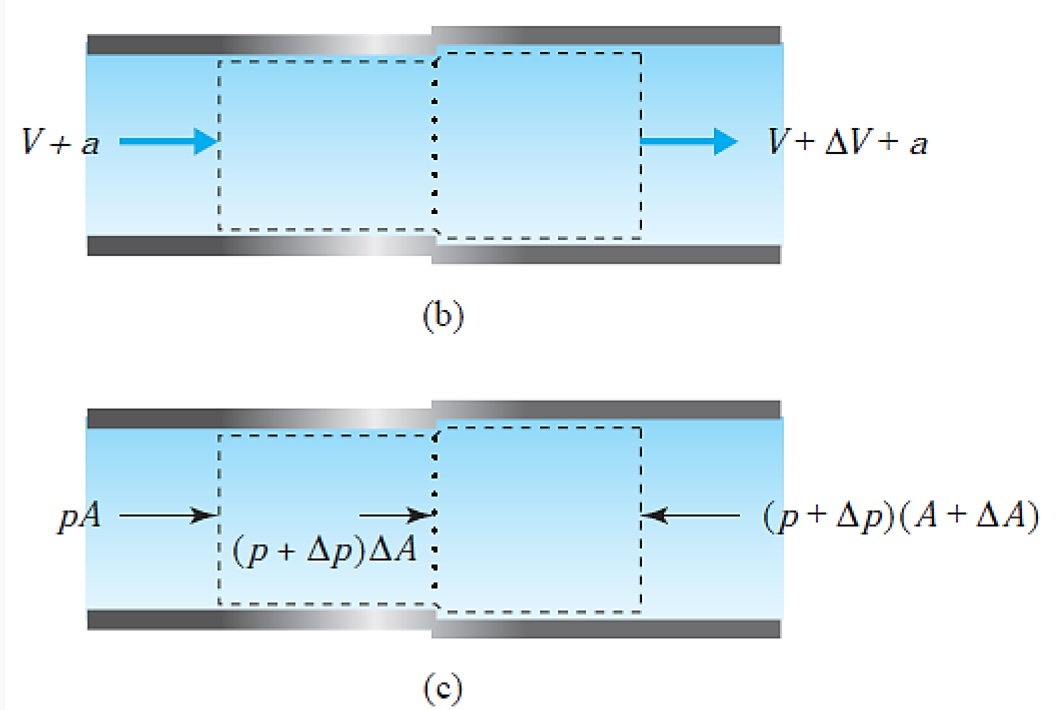
Once the wave has passed through the control volume, the altered conditions p + Δp, V + ΔV, A + ΔA, and ρ + Δρ will persist until the wave reflects from the upstream boundary.

Equations 11.5.10 and 11.5.11 are combined, again recognizing that V << a, and eliminating ΔV, we get,

$$\frac{\Delta p}{\rho a^2} = \frac{\Delta \rho}{\rho} + \frac{\Delta A}{A} \qquad (11.5.13)$$

From the definition of the **bulk modulus of elasticity B for a fluid**, we can relate the change in density to the change in pressure as **$\Delta\rho/\rho = \Delta p/B$.**

The change in pipe area can be related to the change in pressure by considering an instantaneous, elastic response of the pipe wall to pressure changes. Assuming a circular pipe cross section of radius r, we have **$\Delta A/A = 2\Delta r/r$**, and the change in circumferential strain ε in the pipe wall is **$\Delta\varepsilon = \Delta r/r$.**



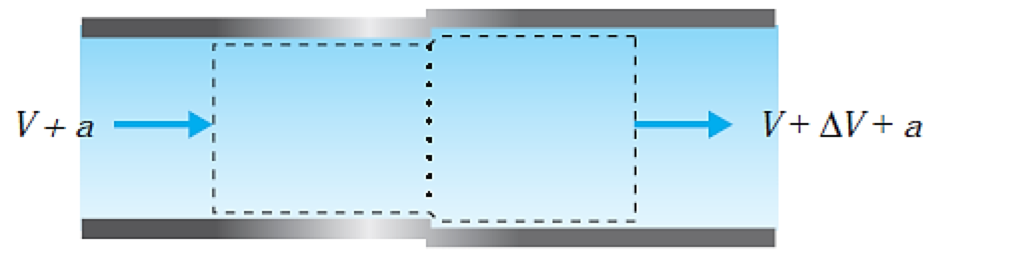
Compressible Flow in an Elastic Pipe

For a **thin-walled pipe** whose thickness e is much smaller than the radius, that is, e << r, the circumferential stress is given by σ = pr/e. For small changes in r and e, **$\Delta\sigma \approx (r/e)\Delta p$**. The elastic modulus for the pipe wall material is the change in stress divided by the change in strain, or

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} \approx \frac{(r/e)\Delta p}{\Delta r/r} = \frac{(2r/e)\Delta p}{\Delta A/A} \qquad (11.5.14)$$

Solving for ΔA/A, and substituting the result into Eq. 11.5.13, along with the relative change in density related to the change in pressure and bulk modulus, there results

$$\frac{\Delta\rho}{\rho a^2} = \frac{\Delta p}{B} + \frac{2r\Delta p}{E} \qquad (11.5.15)$$



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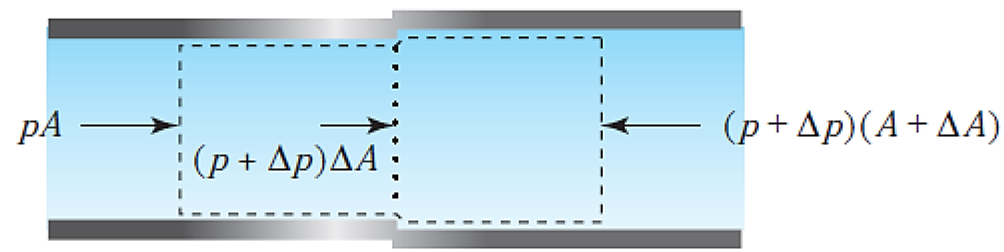


(c)

Compressible Flow in an Elastic Pipe

The parameter Δp can be eliminated in Eq. 11.5.15, the diameter substituted for the radius, r and the relation solved for a:

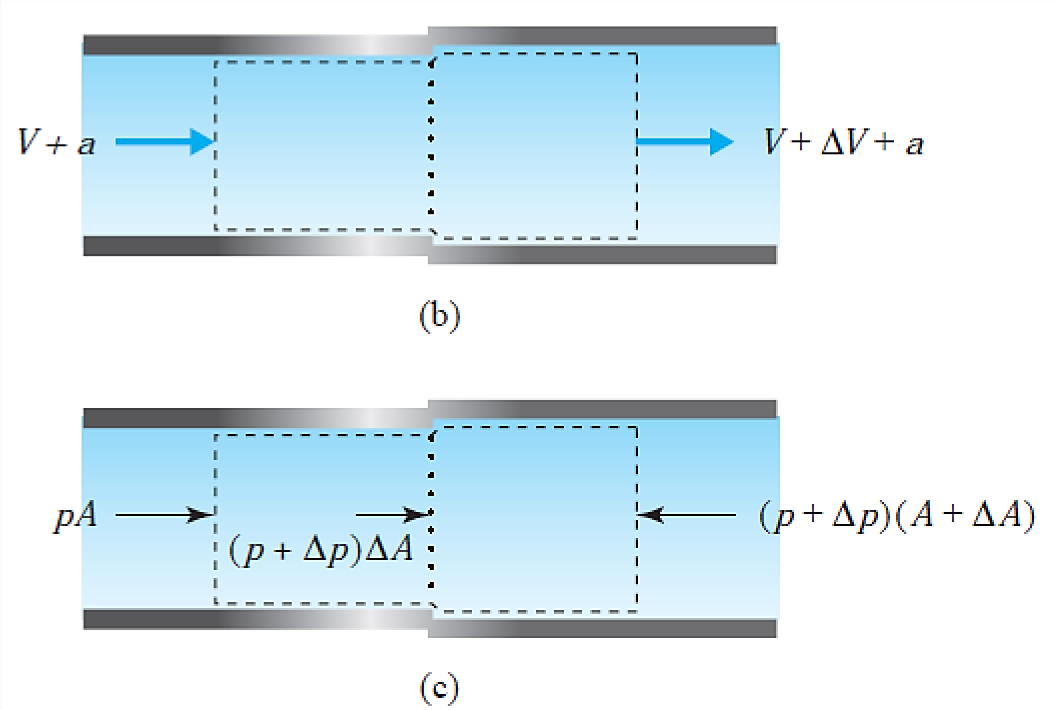
$$a = \sqrt{\frac{B/\rho}{1 + (D/e)(B/E)}} \qquad (11.5.16)$$

Hence the pressure pulse wave speed is shown to be related to the properties of the liquid (ρ and B) and those of the pipe wall (*D*, *e*, and *E*).

If the pipe is very rigid, or stiff, then the term DB/eE << 1, and Eq. 11.5.16 becomes

$$a = \sqrt{B/\rho},$$

which is the speed of sound in an unbounded liquid (see Eq. 1.5.12). Thus the pipe elasticity reduces the speed of the pressure wave.



Compressible Flow in an Elastic Pipe

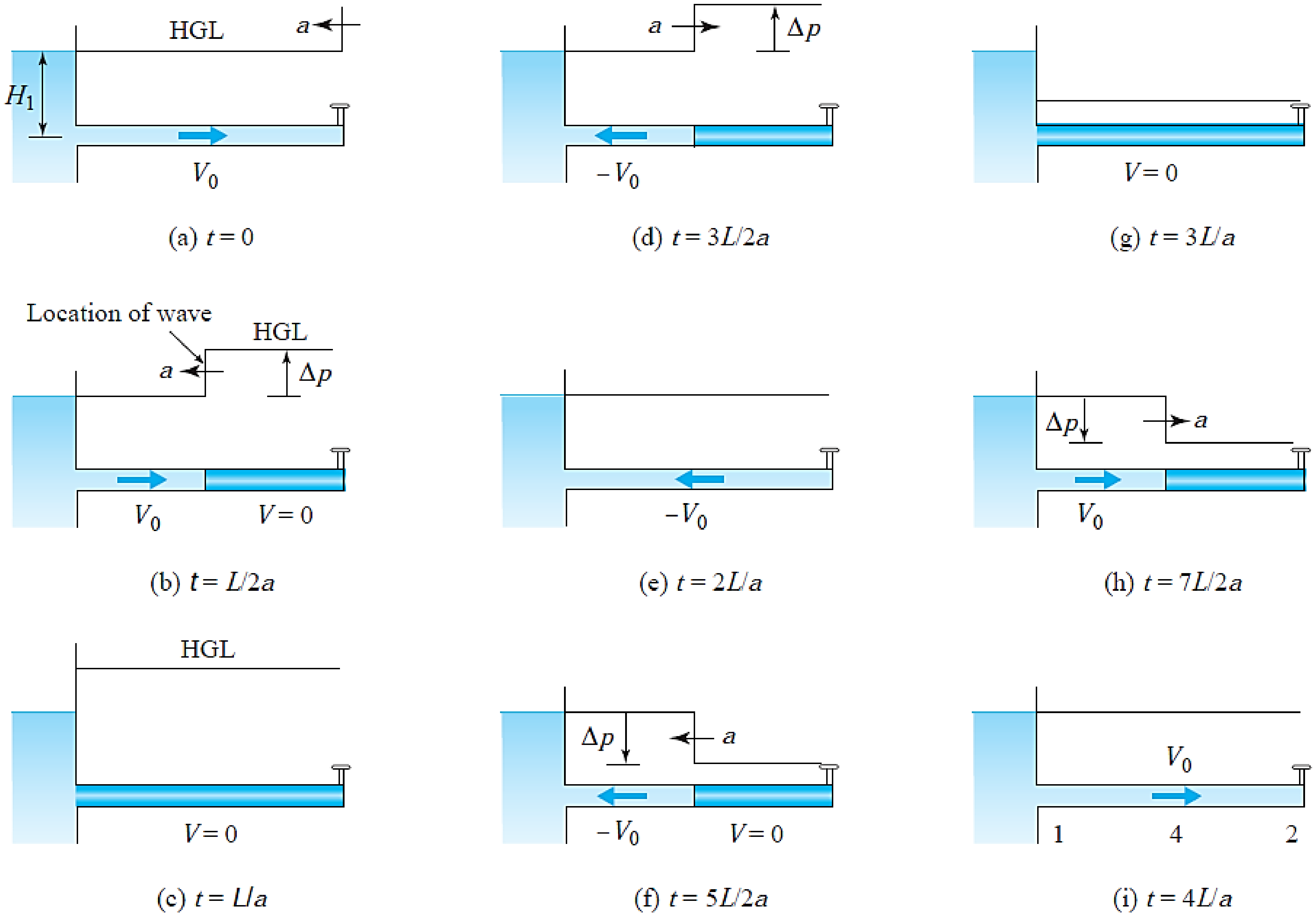


Fig. 11.10 One cycle of wave motion in a pipe due to a sudden valve closure.

Compressible Flow in an Elastic Pipe

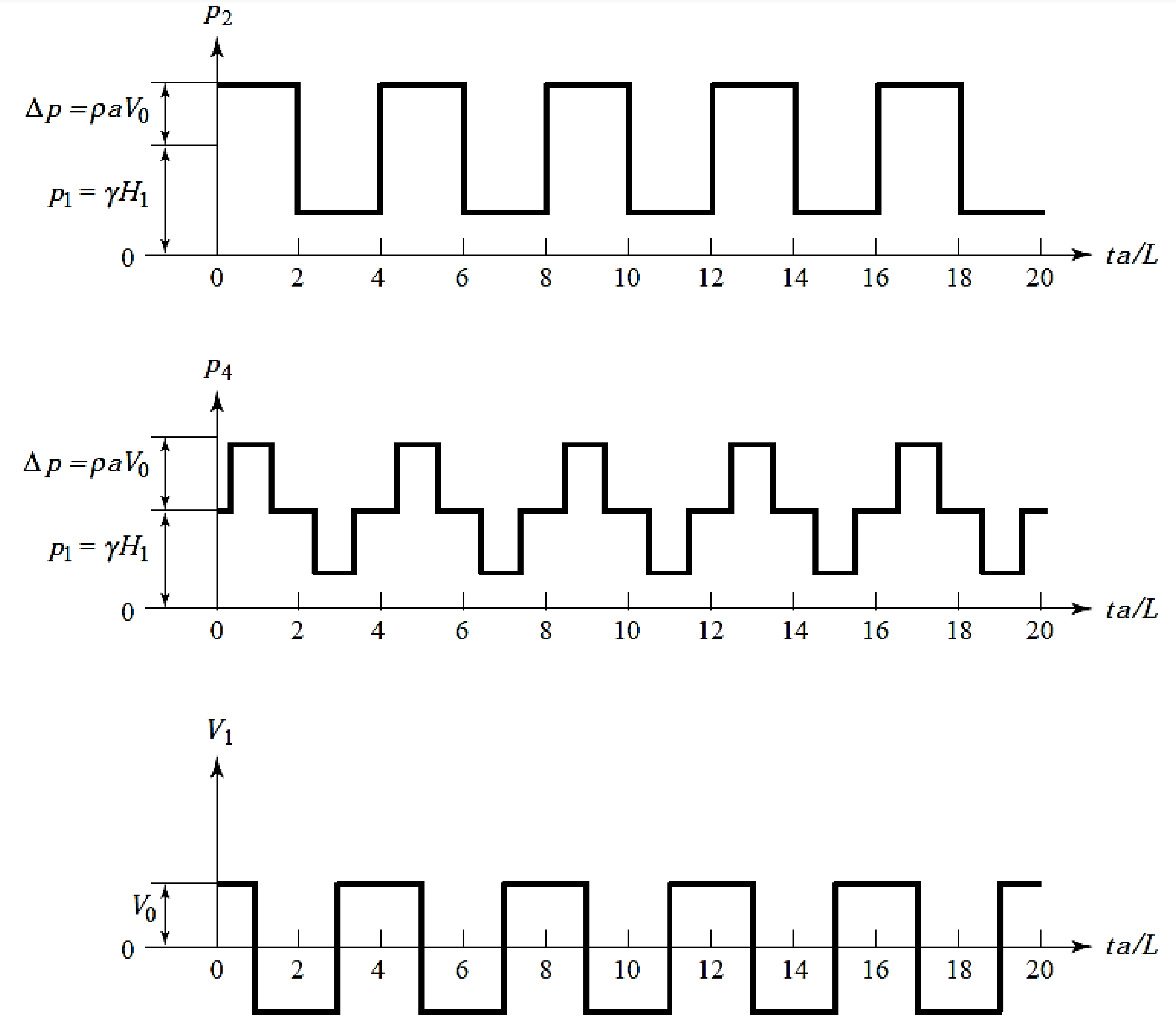
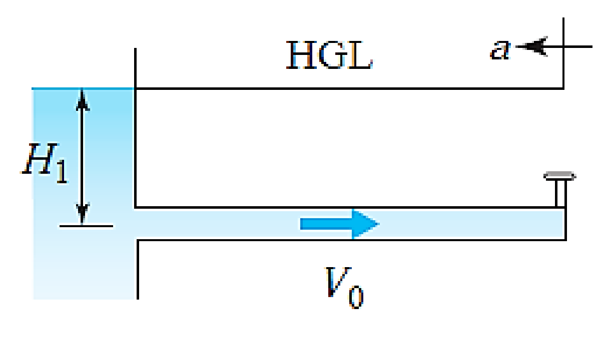


Fig. 11.11 Pressure waveforms at the valve (p_2), pipe midpoint (p_4), and velocity waveform at the entrance to the pipe (V_1).



(a) $t = 0$

Compressible Flow in an Elastic Pipe

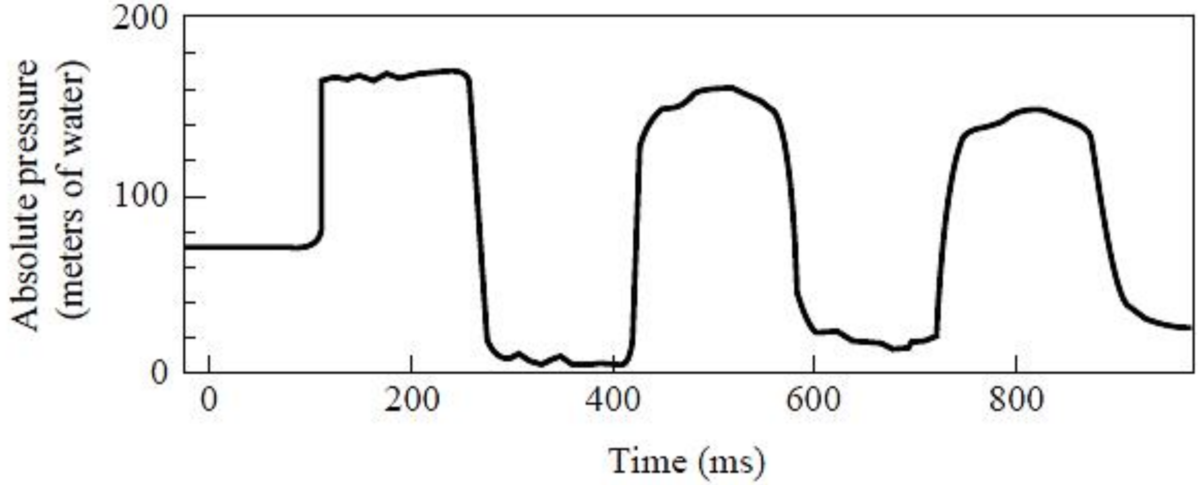
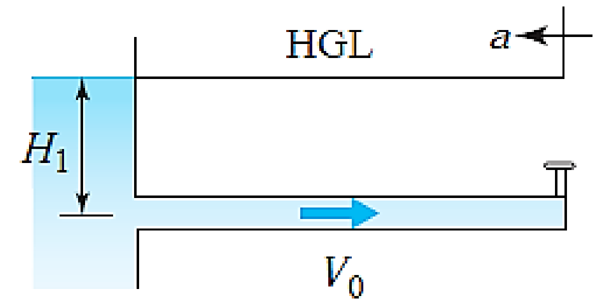


Fig. 11.12 Pressure waveform at the valve for an actual pipe system following rapid valve closure. (After Martin, 1983.)(Martin, C. D., Experimental Investigation of Column Separation with Rapid Valve Closure, Proceedings, 4th International Conference on Pressure Surges, BHRA Fluid Engineering, Cranfield, England, 1983, pp. 77–88. Reproduced with permission.)



(a) $t = 0$

Compressible Flow in an Elastic Pipe

Example 11.11 (Potter)

A steel pipe (E = 207 GPa, L = 1500 m, D = 300 mm, e = 10 mm) conveys water at 20°C. The initial velocity is V_0 = 1 m/s. A valve at the downstream end is closed so rapidly that the motion is considered to be instantaneous, reducing the velocity to zero. Determine the pressure pulse wave speed in the pipe, the speed of sound in an unbounded water medium, the pressure rise at the valve, the time it takes for the wave to travel from the valve to the reservoir at the upstream end, and the period of oscillation.

Solution

The density and bulk modulus of water at 20°C are found in Table B.1: $\rho = 998\ \text{kg/m}^3$ and $B = 220 \times 10^7$ Pa. The pressure wave speed, a, is given by Eq. 11.15.16:

$$a = \sqrt{\frac{220 \times 10^7/998}{1 + \dfrac{0.3}{0.01} \times \dfrac{220 \times 10^7}{207 \times 10^9}}} = 1290\ \text{m/s}$$

The speed of sound in an unbounded water medium is found by use of Eq. 1.5.12 to be

$$a = \sqrt{\frac{220 \times 10^7}{998}} = 1485\ \text{m/s}$$

Note that the sound speed is about 15% larger than the pressure wave speed. To compute the pressure rise at the valve, we recognize that the reduction in velocity is $-V_0 = -1$ m/s. Using Eq. 11.15.12, the increase in pressure upstream of the valve is

$$\Delta p = -\,998 \times 1290 \times (-\,1) = 1.29 \times 10^6\ \text{Pa} \quad \text{or} \quad 1290\ \text{kPa}$$

The wave travel time from the valve to the reservoir is $L/a = 1500/1290 = 1.16$ s, and the period of oscillation is $4L/a = 4 \times 1.16 = 4.65$ s.